Common Core Math in 5th Grade

Fifth grade in the Common Core is the last year with arithmetic as a focus, though in later grades there will be plenty of opportunity to continue practicing these skills — for example, dividing numbers when computing proportions.

This year students will learn to add fractions with unlike denominators. This is a complicated process, and some curricula even suggest using elaborate gimmicks to remember it. In the Common Core approach, students will have a firm grounding in the number line, in renaming fractions ($\frac{!}{!}$ is also $\frac{!}{!}$!), and in adding fractions with the same denominator ($\frac{!}{!} + \frac{!}{!} = \frac{!\,!}{!\,!}$). All of this will make addition of fractions a process that makes sense, rather than something to remember using tricks which use pictures of X's or butterfly wings.

This type of reasoning also helps to apply fraction arithmetic correctly. Many of us remember that you "multiply across" to multiply $\frac{!}{!} \times \frac{!}{!}$, but struggle to know if one *should* multiply in a real world context. A key is that $\frac{!}{!} \times \frac{!}{!}$ is what you get when you split $\frac{!}{!}$ of something into three equal pieces and take two of those. Students will use pictures to reason about problems, as many good problem-solvers often do. From these they will be able to know whether to multiply or divide, and have a sense for what they expect in a reasonable answer.

Students will use similar reasoning about whole numbers and decimals — using sketches, examples, and properties which have been carefully developed, so these arithmetic skills will provide a strong base for algebra.

Examples:

Video Game Scores (see reverse).
In this task, students connect a "real-life" situation to arithmetic with many steps. The students don't have to make calculations, though a teacher could ask them to if necessary. The more important part of the activity is to have students work on their mathematical language skills to interpret expressions in the context of the problem. This gives some great practice leading up to using variables as in algebra. One can just change the task a bit — an unknown amount of bonus points, for example — and it is a good algebra activity.

Tips for parents:

- It is likely that your child is learning in a way you didn't, so you can't just figure out in a minute what's going on. This presents a great opportunity: ask your child to explain some math to you! Communicating reasoning is a skill we want children to have, and it rarely happens enough.
- Children at this point will likely have a strong sense of how "good" they are at math, usually based on how quickly they can calculate. Challenge this! Many of the best mathematicians are slow at calculation, but take time to truly understand a problem. Understanding will eventually be a struggle for everyone in some math class. Just as a musician doesn't expect to play every new piece well, a math learner won't understand every concept right away but can progress until they get there.
- High achievers may be ready to use variables to more deeply reflect on the arithmetic they learn. If they see exactly why three fourths and one half makes five fourths (on the number line, especially), and similarly nine fourths and one half makes eleven fourths and so on, then they could also say that *n* fourths and one half makes *n* + 2 fourths. In symbols, that's $\frac{!}{!} + \frac{!}{!} = !\frac{!}{!} + \frac{!}{!} = !\frac{!\,!\,!}{!}$. This deeper reflection on fraction arithmetic is much more beneficial than rushing though the rules of arithmetic on an accelerated track.

Developed by Tricia Bevans and Dev Sinha of the
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For further information see:
ime.math.arizona.edu/progressions
and illustrativemathematics.org

Example: Video Game Scores

https://www.illustrativemathematics.org/illustrations/590

Eric is playing a video game. At a certain point in the game, he has 31500 points. Then the following events happen, in order: He earns 2450 additional points. He loses 3310 points. The game ends, and his score doubles.

> Write an expression for the number of points Eric has at the end of the game. Do not evaluate the expression. The expression should keep track of what happens in each step listed above.
>
> Eric's sister Leila plays the same game. When she is finished playing, her score is given by the expression 3(24500+3610) − 6780. Describe a sequence of events that might have led to Leila earning this score.

Commentary:

Standard 5.OA.2 asks students to "Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them." This task asks students to exercise both of these complementary skills, writing an expression in part A and interpreting a given expression in part B. The numbers given in the problem are deliberately large and "ugly" to discourage students from calculating Eric's and Leila's scores. The focus of this problem is not on numerical answers, but instead on building and interpreting expressions that could be entered in a calculator or communicated to another student.

Solution:

a. When Eric earns 2450 additional points, his score becomes 31500 + 2450. When he loses 3310 points, his score becomes (31500 + 2450) – 3310. (Note that this can also be written without the parentheses.) When Eric's score doubles, the score becomes 2 x ((31500 + 2450) – 3310), which can also be written 2(31500 + 2450 – 3310).

b. Here is a possible sequence of events that might lead to the score given: At a certain point in the game, Leila has 24500 points. She earns 3610 additional points. Her score triples. She loses 6780 points.

c. Note that the order of the steps is important; rearranging the steps will likely lead to a different expression and a different final score.

Common Core Math in 6th Grade

In sixth grade different number and arithmetic concepts come together and are used in interesting ways. Students are going to use their knowledge of multiplication and division to understand problems involving ratios and proportions. They'll increase their skill with fractions to include dividing fractions. And they'll begin to use equations and expressions with variables. Along the way, they'll also fill in the number line with another type of number as they begin to understand and work with negative numbers.

These topics are all highly interrelated. Students will use tables, graphs, number lines, and diagrams to represent a situation with ratios as different approaches to problem solving and to highlight different structure. For example, suppose a juice blend uses 5 cups of grape juice for every 2 cups of peach juice. A student might produce the following table:

2	4	6	8	10
5	10	15	20	25

Using this they might be able to figure out how many cups of grape juice for 7 cups of peach juice. Graphing these pairs on a coordinate plane would show further structure, such as seeing 2.5 cups of grape juice for every cup of peach juice, and prompt further insights. If students do "cross multiply" to solve ratio problems, it will be a result of a solid understanding of the meaning of ratios.

Geometry in this grade provides some great opportunities. Students will reason about formulas for length, area and volume, and by doing so reinforce their work with equations and expressions, which are new in this grade. At this level, a wide range of applications also opens up.

Examples:

Security Camera (see reverse)

This example gives a sense of how students might get to tie together several different skills in a single situation. For this task students must work with fractions, reason about areas and shapes, calculate percentages — all in a context that has some grounding in the real world. It also highlights the mathematical practices that are so important. This is not a problem that's a breeze-through if you understood the examples in the text. Children are going to have to do some reasoning. They'll have to stick with it. They'll have to communicate why they know they've found the *best* answer. It is possible for nearly every student to begin working on the problem, but there are many opportunities for pushing children beyond the original problem if they are ready for that too. (Is putting the cameras at grid lines realistic? Does our answer change if we don't have to do that?)

Tips for parents:

- Be patient if your child struggles, especially if math has been relatively easy in the past. Make sure to emphasize that this struggle is not an indication of failure and mistakes are just opportunities to learn. See for example: https://www.khanacademy.org/about/blog/post/95208400815/the-learning-myth-why-ill-never-tell-my-son-hes

- Continue to have your child practice math as it comes up in your everyday interactions. (e.g. If it has taken us 3 hours to get two thirds of the way to the cabin, how long do you expect the whole trip will take? Will I have enough money to get 2 pairs of pants and 3 shirts?)

- Ask your child to notice assumptions you make to solve everyday problems with math. For example, if 6 oz. costs \$3.25, how much will 15 oz. cost? Multiplying the cost by 2½ assumes that you *can* purchase 15 oz., and that the unit price is the same for larger quantities.

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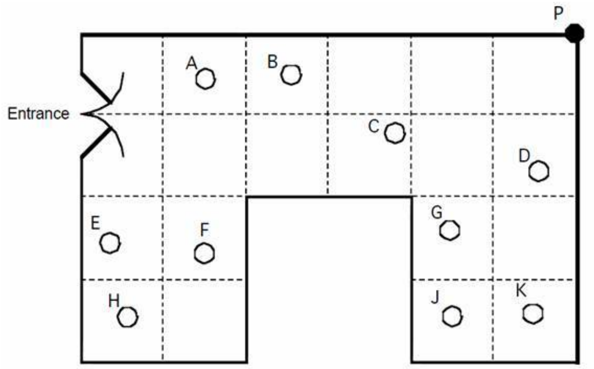
Example: Security Camera

https://www.illustrativemathematics.org/illustrations/115

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. Below is a picture of the shop floor plan with a square grid. The camera can rotate 360°. The shop owner places the camera at point P, in the corner of the shop.



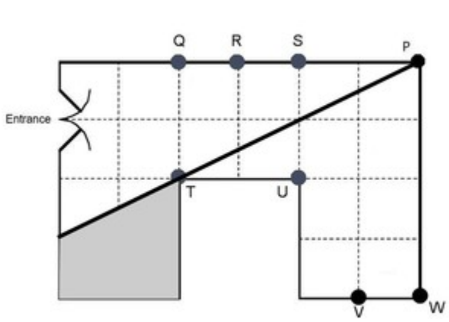
1. The plan shows where ten people are standing in the shop. They are labeled A, B, C, D, E, F, G, H, J, K. Which people cannot be seen by the camera at P?
2. What percentage of the shop is hidden from the camera? Explain or show work.
3. The shopkeeper has to hang the camera at the corners of the grid. Show the best place for the camera so it can see as much of the shop as possible. Explain how you know that this is the best place to put the camera.

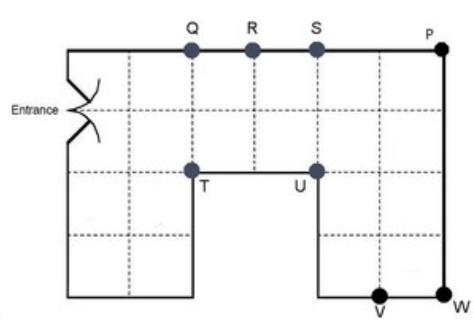
Commentary:

The last question has more than one answer, in the sense that there are three spots that could be considered "best." These three locations all cover the same amount of the store while at the same time miss less of the store than all other possible spots.

Solutions:

1. With the camera at point P, shoppers F and H are hidden from the camera.
2. There are 20 squares on the grid. If a line is drawn from point P to point T and beyond, the region that is hidden from the camera has an area of 3 squares (this region is composed of a triangle with an area of 1 square and a rectangle with an area of 2 squares; see the figure below). There are a total of 17 out of 20 squares visible from point P. 17/20 =0.85, so 85% of the store is visible, and 15% of the store is hidden from point P.
3. Looking at the figure below, the best places to place the camera are Q, R, and S.





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